The maximum and minimum commands on a TI-82 or TI-83 calculator provide another method for finding extreme values of functions. We use this method in the next example.

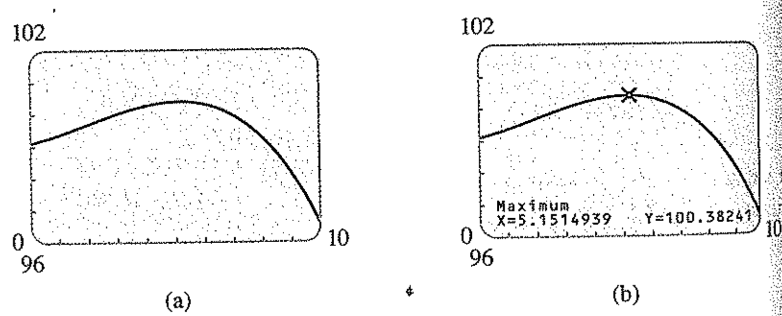
Example 7 A Model for the Food Price Index

A model for the food price index (the price of a representative "basket" of foods) between 1990 and 2000 is given by the function

$$I(t) = -0.0113t^3 + 0.0681t^2 + 0.198t + 99.1$$

where t is measured in years since midyear 1990, so $0 \le t \le 10$, and $I(t)$ is scaled so that $I(3) = 100$. Estimate the time when food was most expensive during the period 1990–2000.

Solution The graph of I as a function of t is shown in Figure 8(a). There appears to be a maximum between $t = 4$ and $t = 7$. Using the maximum command, as shown in Figure 8(b), we see that the maximum value of I is about 100.38, and it occurs when $t \approx 5.15$, which corresponds to August 1995.



(a) (b)

Figure 8

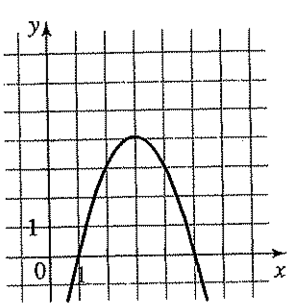
2.5 Exercises

1–4 ■ The graph of a quadratic function f is given.

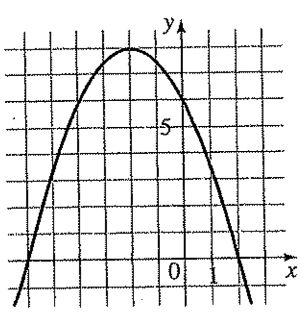
(a) Find the coordinates of the vertex.

(b) Find the maximum or minimum value of f.

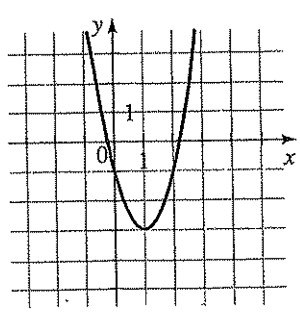
1. $f(x) = -x^2 + 6x - 5$



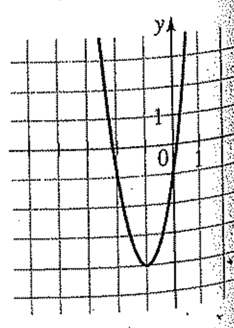
2. $f(x) = -\frac{1}{2}x^2 - 2x + 6$



3. $f(x) = 2x^2 - 4x - 1$



4. $f(x) = 3x^2 + 6x - 1$



5–18 ▪ A quadratic function is given.
(a) Express the quadratic function in standard form.
(b) Find its vertex and its x- and y-intercept(s).
(c) Sketch its graph.

5. $f(x) = x^2 - 6x$
6. $f(x) = x^2 + 8x$
7. $f(x) = 2x^2 + 6x$
8. $f(x) = -x^2 + 10x$
9. $f(x) = x^2 + 4x + 3$
10. $f(x) = x^2 - 2x + 2$
11. $f(x) = -x^2 + 6x + 4$
12. $f(x) = -x^2 - 4x + 4$
13. $f(x) = 2x^2 + 4x + 3$
14. $f(x) = -3x^2 + 6x - 2$
15. $f(x) = 2x^2 - 20x + 57$
16. $f(x) = 2x^2 + x - 6$
17. $f(x) = -4x^2 - 16x + 3$
18. $f(x) = 6x^2 + 12x - 5$

19–28 ▪ A quadratic function is given.
(a) Express the quadratic function in standard form.
(b) Sketch its graph.
(c) Find its maximum or minimum value.

19. $f(x) = 2x - x^2$
20. $f(x) = x + x^2$
21. $f(x) = x^2 + 2x - 1$
22. $f(x) = x^2 - 8x + 8$
23. $f(x) = -x^2 - 3x + 3$
24. $f(x) = 1 - 6x - x^2$
25. $g(x) = 3x^2 - 12x + 13$
26. $g(x) = 2x^2 + 8x + 11$
27. $h(x) = 1 - x - x^2$
28. $h(x) = 3 - 4x - 4x^2$

29–38 ▪ Find the maximum or minimum value of the function.

29. $f(x) = x^2 + x + 1$
30. $f(x) = 1 + 3x - x^2$
31. $f(t) = 100 - 49t - 7t^2$
32. $f(t) = 10t^2 + 40t + 113$
33. $f(s) = s^2 - 1.2s + 16$
34. $g(x) = 100x^2 - 1500x$
35. $h(x) = \frac{1}{2}x^2 + 2x - 6$
36. $f(x) = -\dfrac{x^2}{3} + 2x + 7$
37. $f(x) = 3 - x - \frac{1}{2}x^2$
38. $g(x) = 2x(x - 4) + 7$

39. Find a function whose graph is a parabola with vertex $(1, -2)$ and that passes through the point $(4, 16)$.

40. Find a function whose graph is a parabola with vertex $(3, 4)$ and that passes through the point $(1, -8)$.

41–44 ▪ Find the domain and range of the function.

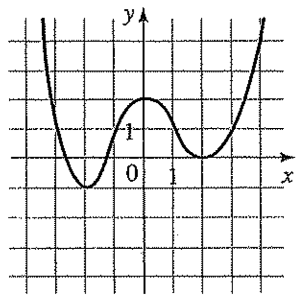
41. $f(x) = -x^2 + 4x - 3$
42. $f(x) = x^2 - 2x - 3$
43. $f(x) = 2x^2 + 6x - 7$
44. $f(x) = -3x^2 + 6x + 4$

45–46 ▪ A quadratic function is given.
(a) Use a graphing device to find the maximum or minimum value of the quadratic function f, correct to two decimal places.
(b) Find the exact maximum or minimum value of f, and compare with your answer to part (a).

45. $f(x) = x^2 + 1.79x - 3.21$
46. $f(x) = 1 + x - \sqrt{2}x^2$

47–50 ▪ Find all local maximum and minimum values of the function whose graph is shown.

47.



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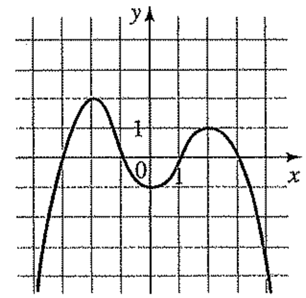

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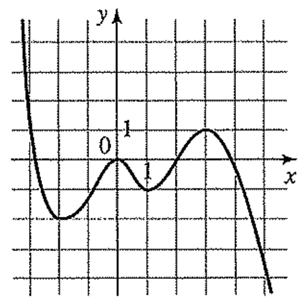
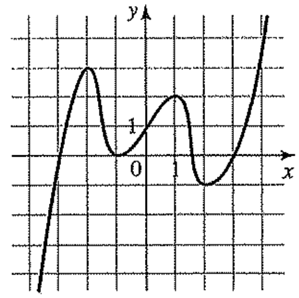


50.



51–58 ▪ Find the local maximum and minimum values of the function and the value of x at which each occurs. State each answer correct to two decimal places.

51. $f(x) = x^3 - x$
52. $f(x) = 3 + x + x^2 - x^3$
53. $g(x) = x^4 - 2x^3 - 11x^2$
54. $g(x) = x^5 - 8x^3 + 20x$
55. $U(x) = x\sqrt{6 - x}$
56. $U(x) = x\sqrt{x - x^2}$
57. $V(x) = \dfrac{1 - x^2}{x^3}$
58. $V(x) = \dfrac{1}{x^2 + x + 1}$

Applications

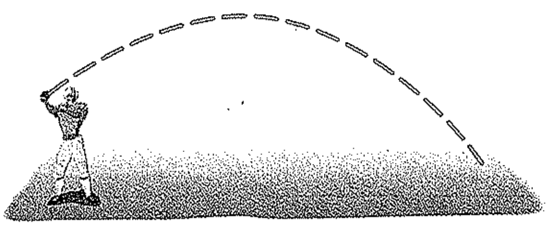
59. **Height of a Ball** If a ball is thrown directly upward with a velocity of 40 ft/s, its height (in feet) after t seconds is given by $y = 40t - 16t^2$. What is the maximum height attained by the ball?

60. **Path of a Ball** A ball is thrown across a playing field. Its path is given by the equation $y = -0.005x^2 + x + 5$,

where x is the distance the ball has traveled horizontally, and y is its height above ground level, both measured in feet.

(a) What is the maximum height attained by the ball?

(b) How far has it traveled horizontally when it hits the ground?



61. **Revenue** A manufacturer finds that the revenue generated by selling x units of a certain commodity is given by the function $R(x) = 80x - 0.4x^2$, where the revenue $R(x)$ is measured in dollars. What is the maximum revenue, and how many units should be manufactured to obtain this maximum?

62. **Sales** A soft-drink vendor at a popular beach analyzes his sales records, and finds that if he sells x cans of soda pop in one day, his profit (in dollars) is given by

$$P(x) = -0.001x^2 + 3x - 1800$$

What is his maximum profit per day, and how many cans must he sell for maximum profit?

63. **Advertising** The effectiveness of a television commercial depends on how many times a viewer watches it. After some experiments an advertising agency found that if the effectiveness E is measured on a scale of 0 to 10, then

$$E(n) = \tfrac{2}{3}n - \tfrac{1}{90}n^2$$

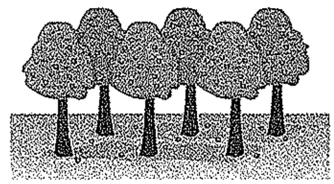
where n is the number of times a viewer watches a given commercial. For a commercial to have maximum effectiveness, how many times should a viewer watch it?

64. **Pharmaceuticals** When a certain drug is taken orally, the concentration of the drug in the patient's bloodstream after t minutes is given by $C(t) = 0.06t - 0.0002t^2$, where $0 \le t \le 240$ and the concentration is measured in mg/L. When is the maximum serum concentration reached, and what is that maximum concentration?

65. **Agriculture** The number of apples produced by each tree in an apple orchard depends on how densely the trees are planted. If n trees are planted on an acre of land, then each tree produces $900 - 9n$ apples. So the number of apples produced per acre is

$$A(n) = n(900 - 9n)$$

How many trees should be planted per acre in order to obtain the maximum yield of apples?

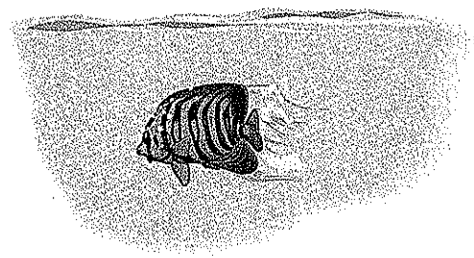


66. **Migrating Fish** A fish swims at a speed v relative to the water, against a current of 5 mi/h. Using a mathematical model of energy expenditure, it can be shown that the total energy E required to swim a distance of 10 mi is given by

$$E(v) = 2.73v^3\frac{10}{v - 5}$$

Biologists believe that migrating fish try to minimize the total energy required to swim a fixed distance. Find the value of v that minimizes energy required.

NOTE This result has been verified; migrating fish swim against a current at a speed 50% greater than the speed of the current.



67. **Highway Engineering** A highway engineer wants to estimate the maximum number of cars that can safely travel a particular highway at a given speed. She assumes that each car is 17 ft long, travels at a speed s, and follows the car in front of it at the "safe following distance" for that speed. She finds that the number N of cars that can pass a given point per minute is modeled by the function

$$N(s) = \frac{88s}{17 + 17\left(\dfrac{s}{20}\right)^2}$$

At what speed can the greatest number of cars travel the highway safely?

68. **Volume of Water** Between 0°C and 30°C, the volume V (in cubic centimeters) of 1 kg of water at a temperature T is given by the formula

$$V = 999.87 - 0.06426T + 0.0085043T^2 - 0.0000679T^3$$

Find the temperature at which the volume of 1 kg of water is a minimum.

69. **Coughing** When a foreign object lodged in the trachea (windpipe) forces a person to cough, the diaphragm thrusts upward causing an increase in pressure in the lungs. At the same time, the trachea contracts, causing the expelled air to move faster and increasing the pressure on the foreign object. According to a mathematical model of coughing, the velocity v of the airstream through an average-sized person's trachea is related to the radius r of the trachea (in centimeters) by the function

$$v(r) = 3.2(1 - r)r^2, \qquad \tfrac{1}{2} \le r \le 1$$

Determine the value of r for which v is a maximum.

Discovery • Discussion

70. **Maxima and Minima** In Example 5 we saw a real-world situation in which the maximum value of a function is important. Name several other everyday situations in which a maximum or minimum value is important.

71. **Minimizing a Distance** When we seek a minimum or maximum value of a function, it is sometimes easier to work with a simpler function instead.
   (a) Suppose $g(x) = \sqrt{f(x)}$, where $f(x) \ge 0$ for all x. Explain why the local minima and maxima of f and g occur at the same values of x.
   (b) Let $g(x)$ be the distance between the point $(3,0)$ and the point (x,x^2) on the graph of the parabola $y = x^2$. Express g as a function of x.
   (c) Find the minimum value of the function g that you found in part (b). Use the principle described in part (a) to simplify your work.

72. **Maximum of a Fourth-Degree Polynomial** Find the maximum value of the function

$$f(x) = 3 + 4x^2 - x^4$$

[*Hint:* Let $t = x^2$.]

2.6 Modeling with Functions

Many of the processes studied in the physical and social sciences involve understanding how one quantity varies with respect to another. Finding a function that describes the dependence of one quantity on another is called *modeling*. For example, a biologist observes that the number of bacteria in a certain culture increases with time. He tries to model this phenomenon by finding the precise function (or rule) that relates the bacteria population to the elapsed time.

In this section we will learn how to find models that can be constructed using geometric or algebraic properties of the object under study. (Finding models from *data* is studied in the *Focus on Modeling* at the end of this chapter.) Once the model is found, we use it to analyze and predict properties of the object or process being studied.

Modeling with Functions

We begin with a simple real-life situation that illustrates the modeling process.

Example 1 Modeling the Volume of a Box

A breakfast cereal company manufactures boxes to package their product. For aesthetic reasons, the box must have the following proportions: Its width is 3 times its depth and its height is 5 times its depth.

(a) Find a function that models the volume of the box in terms of its depth.
(b) Find the volume of the box if the depth is 1.5 in.
(c) For what depth is the volume 90 in^3?
(d) For what depth is the volume greater than 60 in^3?